Solution Sample

Electromagnetic (I)
1st Midterm Exam.

College of Electronic Technology
Department of Communications

Date: 01/10/2022
Time: 90 min.

Name: ______________________________ :الاسم ورقم القيد

Answer the following questions:

Q1) Consider the following vectors:

$\vec{A} = \hat{a}_x + 2\hat{a}_y - 3\hat{a}_z\,,\quad \vec{B} = -4\hat{a}_y + \hat{a}_z\,,\quad \vec{C} = 5\hat{a}_x - 2\hat{a}_z$

Find:

1. $\vec{a_A}$
2. $\vec{A}.\vec{B}$
3. $|\vec{A} - \vec{B}|$
4. θ_{AB}
5. *Prove that* : $\vec{A}.(\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}).\vec{C}$

① $\vec{a_A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z}{\sqrt{1+4+9}} = \frac{1}{\sqrt{14}}\hat{a}_x + \frac{2}{\sqrt{14}}\hat{a}_y - \frac{3}{\sqrt{14}}\hat{a}_z$

② $\vec{A}.\vec{B} = (-8-3) = -11$

③ $|\vec{A}-\vec{B}| = |\hat{a}_x + 6\hat{a}_y - 4\hat{a}_z| = \sqrt{1+36+16} = \sqrt{53}$

④ $\theta_{AB} = \cos^{-1}\left(\frac{A.B}{|A|\,|B|}\right) = \cos^{-1}\left(\frac{-11}{\sqrt{1+4+9}\sqrt{16+1}}\right) = \cos^{-1}\left(\frac{-11}{\sqrt{14}\sqrt{14}}\right) = \cos^{-1}\left(\frac{-11}{\sqrt{238}}\right) = 135.48^\circ$

⑤ $\vec{B}\times\vec{C} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & -4 & 1 \\ 5 & 0 & -2 \end{vmatrix}$

$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & -3 \\ 0 & -4 & 1 \end{vmatrix}$

$\vec{B}\times\vec{C} = 8\hat{a}_x + 5\hat{a}_y + 20\hat{a}_z$

$\vec{A}\times\vec{B} = -10\hat{a}_x - \hat{a}_y - 4\hat{a}_z$

$\vec{A}.(\vec{B}\times\vec{C}) = 8+10-60 = -42$

$(\vec{A}\times\vec{B}).\vec{C} = -50+8 = -42$

=

#

Q2) Determine the area of a cylindrical surface described by:

$$\rho = 5m, 30^\circ \le \emptyset \le 60^\circ, 0 \le z \le 3m$$

As in the figure Q2

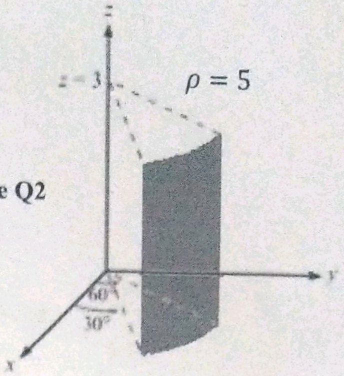


Figure Q2

$\vec{ds} = \rho \, d\phi \, dz \; \hat{a}_\rho$

$\vec{S} = \rho \int_0^3 dz \int_{\pi/6}^{\pi/3} d\phi$

$\vec{S} = (5) \; z \Big|_0^3 \; \phi \Big|_{\pi/6}^{\pi/3} = 5(3)\left[\frac{\pi}{3} - \frac{\pi}{6}\right]$

$\vec{S} = \cancel{15}^{\,5}\left(\frac{\pi}{\cancel{6}_{\,2}}\right) = \frac{5\pi}{2} \, m^2 \;,\; 7.854 m^2$

$\cancel{30}^{\,1}{}^\circ \times \frac{\pi}{\cancel{180}_{\,6}} = \frac{\pi}{6}$

$\cancel{60}^{\,1}{}^\circ \times \frac{\pi}{\cancel{180}_{\,3}} = \frac{\pi}{3}$

Q3) Prove that the vector $\vec{A} = (x + y)\hat{a}_x + (y - x)\hat{a}_y + z\,\hat{a}_z$ in Cartesian coordinates is equal to $\vec{A} = \rho\hat{a}_\rho - \rho\hat{a}_\varphi + z\,\hat{a}_z$ in cylindrical coordinates.

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x+y \\ y-x \\ z \end{bmatrix}$$

$A_\rho = (x+y)\cos(\phi) + (y-x)\sin\phi = \rho(\cos(\varphi)+\sin(\varphi))\cos(\phi) + \rho(\sin(\phi)-\cos(\phi))\sin\phi$

$A_\rho = \rho \sin(\phi)\cos(\phi) + \rho\cos^2(\phi) + \rho\sin^2(\phi) - \rho\sin(\phi)\cos(\phi) = \rho(\sin^2(\phi)+\cos^2(\phi))$

$A_\rho = \rho$

~~$A_\phi = \rho(\cos$~~

$A_\phi = -(x+y)\sin(\phi) + (y-x)\cos(\phi)$

$A_\phi = -\rho\sin(\phi)(\cos(\phi)+\sin(\phi)) + \rho\cos(\phi)(\sin(\phi) - \cos(\phi))$

$A_\phi = -\rho\sin(\phi)\cos(\phi) - \rho\sin^2(\phi) + \rho\sin(\phi)\cos(\phi) - \rho\cos^2(\phi)$

$A_\phi = -\rho(\sin^2(\phi) + \cos^2(\phi)) = -\rho$

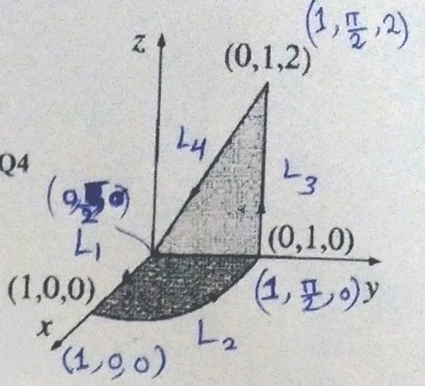
$A_z = z$

$\vec{A} = \rho\,\hat{a}_\rho - \rho\,\hat{a}_\phi + z\,\hat{a}_z$ #

Q4) The curl of the vector $\vec{G} = \rho(2 + \sin^2\varphi)\hat{a}_\rho + \rho\sin(\varphi)\cos(\varphi)\,\hat{a}_\varphi + 3z\,\hat{a}_z$ is **equal to zero.**
Verify Stoke's theorem for the figure shown in Figure Q4.

$\vec{dl} = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$

Figure Q4



For $L_1 \Rightarrow \phi = 0,\ z = 0,\ \rho\,(0 \to 1)$

$\vec{dl} = d\rho\,\hat{a}_\rho$

$\sin(0) = 0$

$\int_{L_1} \vec{A}\cdot\vec{dl} = \int_0^1 (2 + \sin^2\phi)\rho\,.d\rho = 2\int_0^1 \rho\, d\rho = 2\frac{\rho^2}{2}\Big|_0^1 = \underline{\underline{1}}$

For $L_2 \Rightarrow z = 0,\ \rho = 1,\ \phi: 0 \to \frac{\pi}{2}$

$\vec{dl} = \rho\, d\phi\,\hat{a}_\phi$

$\int_{L_2} \vec{A}\cdot\vec{dl} = \int_0^{\frac{\pi}{2}} \rho^2 \sin(\phi)\cos(\phi)\,.d\phi = \rho^2\frac{\sin^2(\phi)}{2}\Big|_0^{\frac{\pi}{2}} = \frac{(1)^2}{2}\left[\sin^2(\tfrac{\pi}{2}) - \sin^2(0)\right] = \underline{\underline{\frac{1}{2}}}$

For L_3 :- $\phi = \frac{\pi}{2},\ \rho = 1,\ z\,(0 \to 2)$

$\vec{dl} = dz\,\hat{a}_z$

$\int_{L_3} \vec{A}\cdot\vec{dl} = 3\int_0^2 z\, dz = 3\frac{z^2}{2}\Big|_0^2 = \frac{3}{2}(4) = \underline{\underline{6}}$

For L_4 :- $\phi = \frac{\pi}{2},\ z: 2 \to 0,\ \rho: 1 \to 0$

$\vec{dl} = d\rho\,\hat{a}_\rho + dz\,\hat{a}_z$

$\int_{L_4} \vec{A}\,\vec{dl} = \int_1^0 \rho(2 + \sin^2\phi))\, d\rho + \int_2^0 3z\, dz$

$= 2 + \sin^2(\phi)\left(\frac{\rho^2}{2}\right)\Big|_1^0 + 3\frac{z^2}{2}\Big|_2^0$

$= (2+1)(-\frac{1}{2}) + \frac{3}{2}(-4) = \frac{-3}{2} - 6 = \underline{\underline{-7.5}}$

$\oint_L \vec{A}\,\vec{dl} = 1 + \frac{1}{2} + 6 - 7.5 = 0$

$\oint_L \vec{A}\cdot\vec{dl} = \int \nabla\times\vec{A}\,\vec{ds} = 0$